

Math 1320: Systems of Inequalities

What is a linear inequality? Previously, we worked with linear equations of the form $Ax + By = C$, whose graphs are straight lines. If we swap out the equals sign for $<$, $>$, \leq , or \geq , then we get a linear inequality, like the example below:

$$x + 2y > 3$$

A system of linear inequalities is when two or more inequalities are working together.

What are the solutions to a system of inequalities? Similar to solutions for systems of linear equations, we are looking for all ordered pairs (x, y) that make all inequalities in our system true. To solve a system of inequalities, we need to graph each inequality, then find the area that all graphs have in common (if there is one). This area is the solution set of all points that satisfy the inequalities.

How do we graph linear inequalities in two variables?

1. Replace the inequality symbol with an equal sign and graph the corresponding linear equation. Draw a solid line if the original inequality contains a \leq or \geq symbol. Draw a dashed line if the original inequality contains a $<$ or $>$ symbol.
2. Choose a test point from one of the half-planes. (Do not choose a point on the line.) Substitute the coordinates of the test point into the inequality.
3. If a true statement results, shade the half-plane containing the test point. If a false statement results, shade the half-plane not containing the test point.

Example 1. Graphing a Linear Inequality in Two Variables

Graph: $x + 2y > 3$

We can also graph inequalities in the form $y > mx + b$ or $y < mx + b$ without using test points. The inequality symbols tells us which side of the line to shade:

- If $y > mx + b$, shade the half-plane above the line $y = mx + b$
- If $y < mx + b$, shade the half-plane below the line $y = mx + b$

Example 2. Graphing a Nonlinear Inequality in Two Variables

Graph: $y \leq x^2 - 4$

Example 3. Graphing a System of Inequalities

Graph the solution set of the system:
$$\begin{cases} x^2 + y^2 < 16 \\ x + y > 1 \end{cases}$$

First, we need to graph both inequalities following the steps from the previous examples.

Practice Problems

1. Graph the inequality: $x^2 - y \geq 1$
2. Graph the inequality: $2x - y < -4$
3. Graph the solution set of the system:
$$\begin{cases} x^2 - y \geq 1 \\ 2x - y < -4 \end{cases}$$

Solutions to Practice Problems

